Digital image processing of moiré fringe patterns with an application to fractures in bovine dentin

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Abstract. A single fringe phase shifting technique is presented for extracting the whole-field displacement distribution from single fringe patterns. By making a small modification in the traditional Fourier transform (FFT) method for fringe processing, the proposed method is capable of maintaining the fringe sign information. An additional approach is introduced that utilizes knowledge of the fringe sign in identifying fringe skeletons with high precision. A combination of these methods of fringe pattern processing was used to examine the whole-field displacement distribution in double cantilever beam (DCB) fracture specimens prepared from bovine maxillary molars. Phase maps and precise fringe skeletons were generated using the proposed techniques from fringe patterns obtained using moiré interferometry. Both aspects of the displacement measurements were used to determine the local displacement distribution near the crack tip, and to study the mechanics of fracture.

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1 Introduction
After decades of effort, recent developments in fringe processing have provided an automated and accurate way to analyze fringe patterns resulting from optical methods of displacement measurement. Although a variety of methods for processing fringe patterns exist, there are two fundamental methods that are predominantly used. One of these methods consists of extracting skeleton lines from the fringe patterns to enable interpolation of the whole-field displacement. The second method is based on phase shifting. Through the use of a phase shifter [e.g., a piezoelectric transducer (PZT)], a known uniform phase is added to the interferometric fringe patterns to obtain a series of fringe pattern images under the same magnitude of applied load. Using specified formulas, the displacement distribution can then be determined from pixel grayscale values. Phase shifting is generally limited to analyses of steady-state displacement fields. When only a single fringe pattern can be documented under a set of prescribed loading conditions, the Fourier transform can be adopted to obtain a phase map for the displacement field. However, due to loss of the fringe direction sign information, Fourier transform analyses have limited application if the fringe direction is unknown.

The objective of this work is to utilize two unique methods of fringe pattern processing in an examination of the displacement field during stable crack growth within double cantilever beam (DCB) fracture specimens. The displacement measurements are conducted using moiré interferometry and the DCB fracture specimens are obtained from the teeth of mature bovine. A modified phase shifting technique is used to perform a quantitative analysis of the whole-field displacement distribution through the development of a phase map. In addition, fringe skeletons are also extracted near the crack tip to ascertain precise displacement information, which is not available from the traditional FFT method.

2 Background
The mechanical properties of biological hard tissues are often needed in the development of engineered materials that are essential for maintaining or restoring lifelong health. For example, a detailed knowledge of the mechanical properties of human teeth is essential for the development of new restorative dental materials. Based on the importance of fracture resistance to the success of restorative dentistry, the fracture toughness ($K_{IC}$) of dentin and enamel, the primary constituents of teeth, has been estimated using various experimental methods.

The mechanical properties of biological hard tissues are generally difficult to evaluate due to the need for preserving the material condition (e.g., hydration, etc.) and size constraint. The fracture properties of teeth can be determined using conventional experimental approaches (e.g., indentation, compact tension tests), but these methods disregard the complex tissue structure and seldom reveal the individual mechanisms contributing to the global material response. Moiré interferometry has recently been used to examine the mechanical properties of teeth and was...
found useful in evaluating the influence of anisotropy and tissue structure on the tooth response. However, interferometric techniques have not been used to study the fracture of dentin or enamel.

In this study an experimental analysis of crack growth in bovine dentin was conducted using moiré interferometry. Fringe fields obtained during the experimental analysis warranted the use of specialized methods of fringe processing for determination of the desired displacement information. Here, details of the experimental analysis are presented and application of the proposed methods for fringe pattern processing is discussed. Results from this analysis will be used to examine the mechanisms of energy dissipation associated with crack growth in bovine dentin using a hybrid approach, which consists of complementary use of both experimental and numerical methods.

3 Fringe Pattern Processing

Two methods of fringe pattern processing were adopted to extract detailed quantitative information from the fringe fields that were documented during stable crack growth in the dentin specimens. Although these methods are applicable to fringe patterns from most optical techniques, they are described here with respect to analyzing fringe patterns resulting from moiré interferometry.

The intensity \( I(r) \) of an optical fringe pattern neglecting the contribution of noise can be expressed as

\[
I(r) = I_0(r) + I_1(r) \cos[\varphi(r)],
\]

where \( I_0(r) \) is the background image intensity, \( I_1(r) \) is the variation in intensity resulting from fringe patterns, and \( \varphi(r) \) represents the phase introduced by deformation. We like to think of \( I_0(r) \) and \( I_1(r) \) as the background brightness and corresponding contrast resulting from the superposed fringe patterns. Equation (1) can also be written in complex form as

\[
f(r) = \psi(r) + i \varphi(r) = M(r) \exp[i \varphi(r)],
\]

where \( \psi(r) \) and \( \varphi(r) \) are the actual (measured) and unknown components of the fringe pattern, respectively; they are also referred to as the real and imaginary components. If \( \varphi(r) \) is obtained or known, the phase distribution can then be obtained according to

\[
\varphi(r) = \arctan \frac{\psi(r)}{\varphi(r)}.
\]

3.1 Modified Fourier Transform Method

In most circumstances, two-dimensional Fourier transforms are performed to obtain the phase-shifted fringe pattern \( [\psi(r)] \) defined by its imaginary output. In performing the Fourier transform, the original fringe pattern \( \psi(r) \) is used as the input. According to properties of the transform in the frequency domain \( \hat{\hat{f}}(u,v) \), the real part is even and the imaginary part is odd. Its power spectrum has a point-symmetric distribution with respect to the zero frequency \((u,v) = (0,0)\), which means the mutually point-symmetric parts of the power spectrum \( \hat{g}(u,v) \) and \( \hat{g}^*(u,v) \) carry the same information. Before performing the inverse operation, a bandpass filter is used to remove half of the frequency domain. Through this operation there is a loss of sign, since the imaginary part of \( \hat{g}(u,v) \) is inverse point symmetric. As it is unknown whether \( \hat{g}(u,v) \) or \( \hat{g}^*(u,v) \) has been eliminated through filtering, a sign error in \( \varphi(r) \) might occur after performing the inverse transform. Therefore, Fourier transform analyses are mostly limited to an examination of monotonous fringe patterns where the fringe sign is known.\(^{10,11}\)

A modified method of phase shifting is now introduced that has the ability to distinguish the fringe orientation. The orientation \( (\alpha) \) of a fringe pattern is perpendicular to the fringe grayscale gradient \( \beta \), which can be defined by

\[
\beta = \arctan \frac{g_x(r)}{g_y(r)},
\]

where \( g_x(r) \) and \( g_y(r) \) represent the gradient components at point \( r \) along the \( x \) and \( y \) axes in the image plane, respectively. For fringe patterns resulting from moiré interferometry, these quantities are obtained from grayscale information according to

\[
g_x(x,y) = \frac{1}{2} (\text{grayscale}_{x+1,y} - \text{grayscale}_{x-1,y}),
\]

\[
g_y(x,y) = \frac{1}{2} (\text{grayscale}_{x,y+1} - \text{grayscale}_{x,y-1}).
\]

Generally, the operation described by Eqs. (5) and (6) is applied after performing noise suppression using a general average or median filter. According to Eq. (4), the fringe direction \( (\alpha) \) can be obtained by adding or subtracting 90 deg from the grayscale gradient \( (\beta) \). But according to Eq. (4) and the definitions for \( g_x(r) \) and \( g_y(r) \) in Eqs. (5) and (6), \( \alpha \) is limited to the range \( 0 \leq \alpha < 180 \) deg, whereas the actual range extends from 0 to 360 deg. Thus, two fringe directions exist for each \( \beta \).

To obtain an unambiguous (or singular) relationship between the grayscale and fringe direction, Bayesian estimation and Markov random-field theory have been implemented to form a quadratic energy function \([U(p)]\). According to statistical theory, the quadratic energy function \([U(p)]\) has to be minimized, and in such case the probability \( (p) \) at each point in the field can be found at the minimum of the quadratic energy function.\(^{12}\) The energy function is given by

\[
U(p) = \sum_{S} \left[ \left( 1 + C_{r_1} \right) \left( p_r - p_s \right)^2 + \left( 1 - C_{r_1} \right) \left( p_r + p_s - 1 \right)^2 \right] + \mu \left( p_{r_0} - 1 \right)^2,
\]

and is defined over the whole image \((S)\). Here, \( r \) and \( s \) are neighboring spatial locations, \( p_r \) and \( p_s \) represent the probability of whether the fringe direction \( y \) equals \( \alpha + 180 \) deg at point \( r \) and \( s \), respectively, \( \mu \) is a large positive number, and \( C_{r_1} = \cos(\alpha_r - \alpha_s) \).

A set of linear equations is defined in terms of \( U(p) \) by setting \( \partial U(p) / \partial p_r = 0 \) at all points in the image. For ex-
ample, at any point $r$, the partial derivative of Eq. (7) is defined at each adjacent point over the neighborhood ($N$) of eight points as

$$2 \sum_{s \in N} \left[ (1 + C_{rs})(p_r - p_s) + (1 - C_{rs})(p_r + p_s - 1) \right] + \delta_{0} \mu (p_{r0} - 1)^2 = 0$$

(8)

where

$$\delta_{r0} = 1; \quad \text{if } r = r_0$$

$$\delta_{r0} = 0; \quad \text{otherwise.}$$

At each pixel location there is a single linear equation defined according to Eq. (8). There are nine unknowns consisting of $p_r$ defined at point $r$, and $p_s$, which is defined at each of the eight neighboring pixels. For an $n \times m$ image, there is a series of equations that is comprised of a total of $n \times m$ equations. The series of equations are solved simultaneously for the probability ($p_r$) at each pixel location. The fringe direction ($\gamma_r$), which has singular relationship between grayscale and direction, can then be obtained according to

$$\gamma_r = \begin{cases} 
\alpha_r, & p_r < 0.05 \\
\alpha_r + 180, & p_r \geq 0.5
\end{cases}$$

An initial starting point ($r_0$) and direction must be chosen to begin the algorithm.

This process of obtaining an unambiguous relationship between the fringe grayscale and direction is referred to as the Gauss-Markov measure field method (GMMFM) and is adopted to extend the range of $\alpha$ to $0 \leq \alpha \leq 360$ deg. With the fringe direction defined for the whole field from Eq. (8), the imaginary output of the inverse Fourier transform would have correct signs if the fringe direction is within $180 - 360$ deg. Therefore, the phase distribution $\phi(r)$ representing the in-plane displacement can be obtained according to Eq. (3).

### 3.2 Fringe Skeletons

According to Eq. (1), the fringe pattern transverse grayscale has a cosine distribution, where the peak and valley points represent the light and dark points of the fringe pattern, respectively. These points could be identified from the locations where the derivative of the grayscale equals zero over a finite neighborhood of pixels in a specific direction. For a digital image, the location of peak (bright) and valley (dark) points defining the fringe center can be found from satisfying the conditions

**Location of peak**

$$\begin{cases} 
\text{grayscale}_{i+1} - \text{grayscale}_{i-1} = 0 \\
\text{grayscale}_{i-2} + \text{grayscale}_{i+2} - 2\text{grayscale}_i < 0
\end{cases}$$

To implement this processing technique to fringe patterns with arbitrary orientation, the fringe direction ($\gamma_r$) with range $0 \leq \gamma_r \leq 360$ deg is also required to avoid the fringe orientation jump. Image noise can be suppressed using average or median filters along the fringe tangent direction. In practice, 1 of 16 possible directions is implemented to represent the local fringe direction, as shown in Figs. 1(a) and 1(b). For a $5 \times 5$ window, the first four directions are shown in Figs. 1(c) through 1(f). Then, along the fringe normal direction, grayscale derivative processing described in Eqs. (10) and (11) is performed to obtain a binary fringe pattern by setting the grayscale at 255 if $\text{grayscale}_{i+1} - \text{grayscale}_{i-1} < 0$, and 0 otherwise.

Note that the algorithm comprised of activities defined by Eqs. (10) and (11) should be performed along a direction normal to the fringe direction ($\gamma_r$) for maximum precision. The grayscale difference at any point ($\text{grayscale}_{i+1} - \text{grayscale}_{i-1}$) obtained for direction 1 should have a sign difference from that of direction 9. Consequently, the derivative-sign binary-fringe can be obtained with no fringe
intensity maximum and minimum. Direction jump, and its boundary then represents the fringe interferometry except for those obtained from electronic speckle pattern processing can be applied to other types of fringe patterns, moiré fringes in this study, the proposed methods for fringe precisely and used in defining skeletons. Although applied to manner the fringe peaks and valleys can be extracted pre-

represent the fringe intensity maximum/minimum. In this decreasing grayscale. The boundaries of the binary fringes then performed transverse to the fringe direction to obtain a derivative process is established, the noise is then removed from the raw image to suppress noise and then to establish the fringe. As long as an unambiguous relationship between grayscale and fringe direction can be established, the noise is then removed from the raw image along the fringe tangent direction. A derivative process is then performed transverse to the fringe direction to obtain a binary fringe with two regions pertaining to increasing/decreasing grayscale. The boundaries of the binary fringes represent the fringe intensity maximum/minimum. In this manner the fringe peaks and valleys can be extracted precisely and used in defining skeletons. Although applied to moiré fringes in this study, the proposed methods for fringe processing can be applied to other types of fringe patterns, except for those obtained from electronic speckle pattern interferometry (based on the large speckle size in relation to speckle noise).

4 Materials and Experimental Methods

Fresh noncarious maxillary molars were obtained from bovine of between 1 and 2 years of age within 12 hours after death. Double cantilever beam (DCB) specimens were machined from teeth obtained from two different animals. Each specimen was obtained from coronal dentin sections, as illustrated in Fig. 2(a). The specimens were machined from the mesial-distal sections as necessary to avoid interference with the pulp canals. As such, the dentin tubule orientation and potential influence of dentin anisotropy on the fracture behavior were not considered. As the primary objective of this preliminary investigation was to develop a technique to examine the mechanisms of fracture in dentin, the influence of anisotropy were not considered but will be examined in future studies.

The specimen geometry incorporated a 54-deg wedge for pin loading and a chevron-notch tip that was sharpened using an abrasive process to promote stable crack initiation and growth. A longitudinal side groove was introduced, as shown in Fig. 2(a), to channel the direction of stable crack growth. The groove was introduced with orientation parallel to the longitudinal axis of the tooth and perpendicular to the mesial-distal axis. Based on the preparation, stable fracture occurred within a central ligament (0.25 mm wide) of 1.6-mm thickness. An example of a completed DCB specimen after preparation is shown in Fig. 2(b). The specimens were kept moist in tap water between the time of sectioning and the experiments.

4.1 Moiré Grating and Optical Bench

A diffraction grating with frequency of 600 lines/mm was placed on the surface of each DCB specimen opposite the surface with longitudinal channel. As the moisture content must be maintained throughout specimen preparation, a modified transfer technique was employed in reference to the standard replication process. Prior to development and transfer of the grating, the submaster mold photoplate was liberally coated with dilute photo flow to assist in uniform mold release. After drying, the photoplate was sputtered with a 200-nm layer of highly reflective aluminum at a rate of 0.4 nm/s. A thin layer of epoxy (PC-10, Measurements Group, Raleigh, NC) was then applied to the sputtered photoplate surface and placed on a flat glass plate covered with teflon (for release). The epoxy film was allowed to dry for 24 hours, after which the submaster mold and glass plate were carefully separated. Through this process a hardened layer of epoxy with approximately 25-μm thickness was formed on the grating backside. The hydrated bovine DCB specimen was then glued to the epoxy layer with cyanacrolate (“superglue”) along the ungrooved surface. As the epoxy film is less permeable than the sputtered aluminum coating, it served as a cohesive medium for bonding the cyanacrolate. After five minutes of curing, the mold was removed and the specimen was ready for testing. According to the specimen orientation relative to the anatomic planes of the tooth, replication of the specimen grating occurred on either the lingual or buccal surface.

The bovine DCB fracture specimens were placed within a rigid fixture for displacement-controlled wedge loading. In this study only the u-field specimen displacement was recorded, which is perpendicular to the direction of crack extension and reflects the opening displacement. The v-field (or vertical displacement) within the specimen was not recorded, as it is much smaller in comparison to the opening mode displacement. The optical system consisted of a 10-mW He-Ne laser, spatial filter, collimating lens, two-beam interferometer, appropriate supplementary mirrors, and a 35-mm camera. A schematic diagram of the optical arrangement is shown in Fig. 3. Interference of the two oblique beams generated a reference grating in front of the specimen of 1200 lines/mm. Moiré fringe patterns were
reflected with a planar mirror to the 35-mm camera for documentation.

4.2 Stable Crack Growth and Fringe Field

Wedge loading of the dentin specimens was applied in increments of 2.3 N through a stainless steel pin of 4.76 mm diam, as illustrated in Fig. 2(c). Moiré fringes were recorded at the peak load of each increment of pin displacement. The crack length, unit of crack extension, and pin displacement were not recorded during the loading process, as the crack position at each load increment was available (and more accurately obtained) from the moiré fringe field. The incremental wedge loading for each specimen continued until the crack resulted in catastrophic fracture of the DCB specimen or discontinued after a sufficient extent of growth was achieved. On average, each test lasted approximately 30 min; the duration of incremental wedge loading for each specimen depended on the extent of stable crack growth achieved. Two specimens were tested in this preliminary study and each experiment consisted of acquiring the load and displacement history during stable crack growth over a series of load increments.

The moiré fringe pattern recorded on the specimen’s surface at each load increment was digitized and then prepared for further processing. The fringe order \( N \) was converted to the corresponding magnitude of displacement \( u \) in the image plane according to Ref. 12,

\[
  u(x,y) = \frac{1}{f} N(x,y),
\]

where \( f \) is the reference grating frequency (lines/mm). Note that the zero fringe order representing the point of zero opening displacement is located at the crack tip, and the magnitude of opening displacement at each load increment was also calculated according to Eq. (12).

5 Results and Discussion

A typical moiré fringe pattern resulting near the crack tip in a bovine DCB specimen at a wedge load of 15.4 N is shown in Fig. 4(a). To obtain the whole-field displacement corresponding to each crack length, the Fourier transform method was utilized and combined with the method described for identifying the fringe direction. A bandpass filter was applied to the FFT of the interference pattern in the frequency domain to remove unwanted components of the spectrum.\(^{13,14}\) The inverse transform of the frequency spectrum is used to determine the real and imaginary components of the fringe pattern as indicated in Eq. (2). During the inverse Fourier transform, the imaginary output is checked by the fringe direction information. For the points that have fringe direction within \( 180 \leq \gamma \leq 360 \text{ deg} \), a sign change is applied to the imaginary part. The phase distribution of the displacement field is obtained according to Eq. (3) and is shown in Fig. 4(b). If the imaginary output is not corrected by the fringe direction information, phase jumps occur where the fringe direction has changed, as shown for the wrapped phase map in Fig. 4(c).\(^{15}\) Note the limitation of applying the conventional FFT is that the grayscale decreases gradually on both sides of the traction-free crack, as evident in Fig. 4(c), and does not correctly represent the displacement symmetry.

Further information of the displacement distribution could be obtained from the phase map in Fig. 4(b) using an unwrapping process as described in Refs. 16 and 17. Some detailed displacement information was lost near the crack tip and traction-free boundary [Fig. 4(b)] due to the averaging process used for noise suppression.

Fringe skeletons were extracted from the fringe fields obtained at each crack length in the DCB specimens to obtain more precise displacement information near the crack tip. To extract skeletons from the fringe pattern in Fig. 4(a), the fringe directions must be established. Two
subareas of the fringe field in Fig. 4(a) on each side of the crack have been selected and are shown in Figs. 5(a) and 5(b). These two areas have the same fringe orientation (horizontal) but different fringe tangent and normal directions. The fringe tangent and normal in these figures have been defined according to the definitions shown in Fig. 1. With fringe directions known, only the pixels along the fringe tangent are selected in performing noise suppression using average and median filters. Then, Eqs. (10) and (11) are implemented strictly according to the fringe normal directions. In Figs. 5(a) and 5(b) the operation is conducted along directions 13 and 5, respectively, as indicated by the arrowhead in each of these figures. The binary fringe boundaries were then obtained and used in defining fringe skeletons; the skeletons for the fringe field in Fig. 4(a) are shown in Fig. 5(c). The nonlinearity in displacement near the traction-free crack surface is clearly evident from the fringe skeletons in this figure. Using the skeletons, a precise distinction of the displacement distribution near the crack tip can be obtained, which is not available from the phase map. Displacements within the region of fringe curvature located near the crack boundary [Fig. 5(c)] are of particular interest in analyzing the mechanisms of fracture in the dentin. The fringe curvature implies that some component of inelastic deformation has occurred adjacent to the crack (similar to a plastic zone in metals). The implications of this observation on the mechanisms of energy dissipation in the fracture of dentin will be examined in future studies.

A contour map of the displacement distribution surrounding the crack tip in Fig. 4(a) is shown in Fig. 6(a). Note that the displacement distribution in this figure was obtained from the unwrapping phase map in Fig. 4(b). The displacement contours do not represent the moiré fringe locations. The local displacement information is then used to determine the near-tip crack opening displacement (COD) during stable fracture within the dentin. An example of the COD determined for the fringe field in Fig. 4(a) is shown in Fig. 6(b). With application of both methods of fringe processing, the whole-field displacement distribution in the dentin DCB fracture specimens was fully defined for each increment of stable crack growth. The displacement fields are currently being used to support a hybrid analysis, consisting of both experiment and numerical methods, to examine the complicated fracture process in this biological material.

6 Conclusion
Two methods of fringe pattern processing were developed and applied in an examination of stable crack growth in double cantilever beam (DCB) fracture specimens machined from mature bovine dentin. The fracture process was documented using moiré interferometry and the whole-field displacement distribution in the dentin DCB fracture specimens was fully defined for each increment of stable crack growth. The displacement fields are currently being used to support a hybrid analysis, consisting of both experiment and numerical methods, to examine the complicated fracture process in this biological material.
direction jump (fringe sign ambiguity), and provided a complete definition of the displacement distribution within the fracture specimens. Fringe skeletons were also obtained to derive precise displacement information from the fringe patterns near the crack tip and along the traction-free surface.

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