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A Limit Symmetry of Modified KdV Equation and Its Applications*

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Abstract In this letter we consider a limit symmetry of the modified KdV equation and its application. The similarity reduction leads to limit solutions of the modified KdV equation. Besides, a modified KdV equation with new self-consistent sources is obtained and its solutions are derived.

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Key words: symmetry, the mKdV equation, symmetry reduction, self-consistent source, Hirota’s method

1 Introduction

It is well known that soliton solutions can be derived by means of variety of approaches, such as the Inverse Scattering Transformation (IST), Darboux transformation, B"acklund transformation, algebraic geometry approach, bilinear method, and so on. From the viewpoint of the IST, N solitons are identified by N distinct eigenvalues or in other words, N distinct simple poles \( k_j \) of transparent coefficient \( 1/a(k) \). In the symmetry approach\cite{1} the classical N-soliton solution is related to a similarity reduction of squared-eigenfunction symmetries.\cite{2–3}

In the letter we consider a new symmetry for the modified KdV (mKdV) equation. The symmetry is related to the known squared eigenfunction symmetry by a limit procedure, and the group invariant solution from the related similarity reduction is a double-pole solution (Refs. [4–5]), which is a limit solution.\cite{6} Besides, squared-eigenfunction symmetries are related to self-consistent sources of soliton equations.\cite{7–8} The limit solution then leads to an mKdV equation with new self-consistent sources. We use Hirota’s bilinear method to get the solutions for this equation.

The letter is organized as follows. In Sec. 2 we discuss the limit symmetry. In Sec. 3 the related similarity reduction is obtained and solved. Section 4 investigates the mKdV equation with new sources and its solutions.

2 A Limit Symmetry of mKdV Equation

The mKdV equation is

\[
\begin{align*}
 \frac{d}{dt} + 6u^2 & u_x + u_{xxx} = 0, \quad (1)
\end{align*}
\]

with Lax pair

\[
\begin{align*}
 \left( \begin{array}{c}
 \phi_1 \\
 \phi_2 
 \end{array} \right)_x = \left( \begin{array}{c}
 -\lambda & u \\
 -u & \lambda 
 \end{array} \right) \left( \begin{array}{c}
 \phi_1 \\
 \phi_2 
 \end{array} \right), \quad \left( \begin{array}{c}
 \phi_1 \\
 \phi_2 
 \end{array} \right)_t = \left( \begin{array}{c}
 -4\lambda^3 - 2\lambda u^2 \\
 -4\lambda^2 u - 2\lambda u_x - u_{xx} - 2u^3 
 \end{array} \right) \left( \begin{array}{c}
 \phi_1 \\
 \phi_2 
 \end{array} \right), \quad \text{(2)}
\end{align*}
\]

where \( \lambda \) is the spectral parameter. It is known that

\[
\sigma = \left( \phi_1^2 + \phi_2^2 \right)_x, \quad \text{(3)}
\]

is a symmetry of the mKdV equation if \( \phi_1 \) and \( \phi_2 \) satisfy (2).

According to the linearity of linear equation which the symmetry satisfies, this symmetry together with another symmetry \( u_x \) introduces a combined symmetry

\[
\begin{align*}
 \left( \begin{array}{c}
 \phi_{1j} \\
 \phi_{2j} 
 \end{array} \right)_x = \left( \begin{array}{c}
 -\lambda_j & u \\
 -u & \lambda_j 
 \end{array} \right) \left( \begin{array}{c}
 \phi_{1j} \\
 \phi_{2j} 
 \end{array} \right), \quad \left( \begin{array}{c}
 \phi_{1j} \\
 \phi_{2j} 
 \end{array} \right)_t = \left( \begin{array}{c}
 -4\lambda_j^3 - 2\lambda_j u^2 \\
 -4\lambda_j^2 u - 2\lambda_j u_x - u_{xx} - 2u^3 
 \end{array} \right) \left( \begin{array}{c}
 \phi_{1j} \\
 \phi_{2j} 
 \end{array} \right), \quad \text{(6)}
\end{align*}
\]

As in Ref. [2], the above expression can be derived from the Gel’fand–Levitan–Machenko equation in the IST procedure,\cite{9} also acts as a starting point of nonlinearization of Lax pair.\cite{10}
In the letter we consider
\[
\hat{\sigma} = \sum_{j=1}^{N} (\phi_{1j} \psi_{1j} + \phi_{2j} \psi_{2j})_x, \tag{7}
\]
where \(\phi_{1j}, \phi_{2j}\) is the wave function in Eq. (6) and
\[
\begin{pmatrix}
\psi_{1j} \\
\psi_{2j}
\end{pmatrix}_x = 
\begin{pmatrix}
-\lambda_j & u \\
-u & \lambda_j
\end{pmatrix}
\begin{pmatrix}
\psi_{1j} \\
\psi_{2j}
\end{pmatrix}
+ 
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\phi_{1j} \\
\phi_{2j}
\end{pmatrix},
\]
\[
\begin{pmatrix}
\psi_{1j} \\
\psi_{2j}
\end{pmatrix}_t = 
\begin{pmatrix}
-4\lambda_j^3 - 2\lambda_j u^2 & 4\lambda_j^3 u - 2\lambda_j u_x + u_{xx} + 2u^3 \\
-4\lambda_j^3 u - 2\lambda_j u_x - u_{xx} - 2u^3 & 4\lambda_j^3 + 2\lambda_j u^2
\end{pmatrix}
\begin{pmatrix}
\psi_{1j} \\
\psi_{2j}
\end{pmatrix}
+ 
\begin{pmatrix}
12\lambda_j^2 + 2u^2 & -8\lambda_j u + 2u_x \\
8\lambda_j u + 2u_x & -12\lambda_j^2 - 2u^2
\end{pmatrix}
\begin{pmatrix}
\phi_{1j} \\
\phi_{2j}
\end{pmatrix}. \tag{8}
\]
By direct verification we find \(\hat{\sigma}\) satisfies
\[
\hat{\sigma}_t = 6u^2\hat{\sigma}_x + 12uu_x\hat{\sigma} + \hat{\sigma}_{xxx}, \tag{9}
\]
which means \(\hat{\sigma}\) is a symmetry of the mKdV equation as well. Since Eq. (8) can be thought as a derivative of Eq. (6), we call \(\hat{\sigma}\) a limit symmetry. For another in Sec. 3 we will see solutions generated from \(\hat{\sigma}\) by a similarity reduction can be looked as limit solutions.

3 Similarity Reduction and Solutions

We consider the combined symmetry
\[
\sigma' = u_x - \sum_{j=1}^{N} (\phi_{1j} \psi_{1j} + \phi_{2j} \psi_{2j})_x, \tag{10}
\]
where \(\phi_{1j}, \phi_{2j}\) solve Eq. (6) and \(\psi_{1j}, \psi_{2j}\) solve the auxiliary system Eq. (8). Then the constraint \(\sigma' = 0\) yields
\[
u = \sum_{j=1}^{N} (\phi_{1j} \psi_{1j} + \phi_{2j} \psi_{2j}). \tag{11}
\]
Since when \(\phi_{kj}, \psi_{kj}\) \((k = 1 or 2)\) satisfy Eqs. (6) and (8), \(u\) defined by Eq. (11) satisfies the mKdV equation (1) automatically, so the constraint system is reduced to
\[
u = \sum_{j=1}^{N} (\phi_{1j} \psi_{1j} + \phi_{2j} \psi_{2j})_x,
\]
\[
\phi_{1j,x} = -\lambda_j \phi_{1j} + u \phi_{2j},
\]
\[
\phi_{2j,x} = -u \phi_{1j} + \lambda_j \phi_{2j},
\]
\[
\phi_{1j,t} = (-4\lambda_j^3 - 2\lambda_j u^2) \phi_{1j},
\]
\[
\phi_{2j,t} = (4\lambda_j^3 u - 2\lambda_j u_x + u_{xx} + 2u^3) \phi_{2j},
\]
\[
\psi_{1j,x} = -\lambda_j \psi_{1j} + u \psi_{2j} + \phi_{1j},
\]
\[
\psi_{2j,x} = -u \psi_{1j} + \lambda_j \psi_{2j} - \phi_{2j},
\]
\[
\psi_{1j,t} = (-4\lambda_j^3 - 2\lambda_j u^2) \psi_{1j},
\]
\[
\psi_{2j,t} = (4\lambda_j^3 u - 2\lambda_j u_x + u_{xx} + 2u^3) \psi_{2j},
\]
\[
\psi_{1j,x} = -\lambda_j \psi_{1j} + u \psi_{2j} + \phi_{1j},
\]
\[
\psi_{2j,x} = -u \psi_{1j} + \lambda_j \psi_{2j} - \phi_{2j},
\]
\[
\psi_{1j,t} = (-4\lambda_j^3 - 2\lambda_j u^2) \psi_{1j},
\]
\[
\psi_{2j,t} = (4\lambda_j^3 u - 2\lambda_j u_x + u_{xx} + 2u^3) \psi_{2j},
\]
It can be written into bilinear forms as \((with \lambda_j = -k_j)\)
\[
D_x \tilde{f} : f = -2i \left( \sum_{j=1}^{N} \tilde{g}_j \tilde{h}_j + \tilde{h}_j g_j \right),
\]
\[
D_x^2 \tilde{f} : f = 0,
\]
\[
D_x \tilde{g}_j \cdot f = k_j g_j \tilde{f},
\]
\[
D_x \tilde{h}_j \cdot f = k_j h_j \tilde{f} + g_j \tilde{f},
\]
\[
(D_x + D_x^3 + 3k_j^2 D_x) g_j \cdot f = 0,
\]
\[
(D_x + D_x^3 + 3k_j^2 D_x) h_j \cdot f + 6k_j D_x g_j \cdot f = 0, \tag{13}
\]
by the dependent variable transformations
\[
u = \sum_{j=1}^{N} (\phi_{1j} \psi_{1j} + \phi_{2j} \psi_{2j})_x,
\]
\[
\phi_{1j} = \tilde{g}_j / \tilde{f} + g_j / \tilde{f},
\]
\[
\phi_{2j} = i \left( \frac{g_j}{\tilde{f}} - \frac{g_j}{\tilde{f}} \right),
\]
\[
\psi_{1j} = \tilde{h}_j / \tilde{f} + h_j / \tilde{f},
\]
\[
\psi_{2j} = i \left( \frac{h_j}{\tilde{f}} - \frac{h_j}{\tilde{f}} \right), \tag{14}
\]
where \(\tilde{f}, \tilde{g}_j, \tilde{h}_j\) are the complex conjugates of \(f, g_j, h_j\), and \(D\) is the well-known Hirota’s bilinear operator defined by\(^{[11]}\)
\[
D_x^m D_x^n a(t,x) \cdot b(t,x) = \frac{\partial^m}{\partial s^n} \frac{\partial^n}{\partial y^n} a(t+s, x+y)b(t-s, x-y)|_{s=0,y=0}, \quad m, n = 0, 1, 2, \ldots
\]
Next we expand \(g_j, h_j, f\) as
\[
f = 1 + \sum_{l=1}^{\infty} f^{(2l)} \varepsilon^{2l}, \quad g_j = \sum_{l=1}^{\infty} g_j^{(2l-1)} \varepsilon^{2l-1}, \quad h_j = \sum_{l=1}^{\infty} h_j^{(2l-1)} \varepsilon^{2l-1}, \tag{15}
\]
and substitute them into Eq. (13). When \( N = 1 \), the expansions can be truncated by taking \( f^{(2l)} = g_1^{(2l-1)} = h_1^{(2l-1)} = 0 \) for \( l \geq 3 \), then we have
\[
f^{(2)} = i \left( x - 12k_1^2t - \frac{1}{2k_1} \right) e^{2\xi_1},
\]
\[
f^{(4)} = \frac{1}{16k_1^2} e^{4\xi_1},
\]
\[
g_1^{(1)} = \sqrt{k_1} e^{\xi_1}, \quad g_1^{(3)} = -i\sqrt{k_1} e^{3\xi_1} \frac{1}{4k_1},
\]
\[
h_1^{(1)} = \sqrt{k_1} e^{\xi_1} (x - 12k_1^2t),
\]
\[
h_1^{(3)} = i\sqrt{k_1} \left( \frac{x}{4k_1} - 3k_1^2t - \frac{1}{4k_1^2} \right) e^{3\xi_1},
\]
\[
f^{(2l)} = g_1^{(2l-1)} = h_1^{(2l-1)} = 0, \quad l \geq 3,
\]
\[
\xi_1 = k_1 x - 4k_1^3 t + \xi_1^{(0)}, \quad (16e)
\]
with real parameters \( k_1, e^{\xi_1^{(0)}} \).

We investigate dynamics of the above solution, i.e.,
\[
f = 1 + i \left( x - 12k_1^2t - \frac{1}{2k_1} \right) e^{2\xi_1} + \frac{1}{16k_1^2} e^{4\xi_1},
\]
\[
u = i \left( \ln f \right)_x = 2 \left( \arctan \frac{x - 12k_1^2t - 1/2k_1}{e^{-2\xi_1} + (1/16k_1^2) e^{2\xi_1}} \right), \quad (17)
\]
where we have taken \( \varepsilon = 1 \) in Eq. (15). As depicted in Fig. 1(a), the solution is smooth and it has two normal solitons with constant-amplitude \( 2|k_1| \). To realize the asymptotic behaviors of such a solution, we put it in the following moving coordinate frame (Fig. 1(b))
\[
(X = x - 4k_1^2 t, t).
\]

By means of asymptotic analysis we find that the four branches in Fig. 1(b) asymptotically are \( (k_1 > 0) \)
\[
(X = \frac{1}{2k_1} (\ln(4k_1) - \ln(-32k_1^3 t)), \quad (t \to -\infty, \quad X \to -\infty),
\]
\[
(X = \frac{1}{2k_1} (\ln(4k_1) + \ln(-32k_1^3 t)), \quad (t \to -\infty, \quad X \to \infty),
\]
\[
(X = \frac{1}{2k_1} (\ln(4k_1) + \ln(32k_1^3 t)), \quad (t \to \infty, \quad X \to \infty),
\]
\[
(X = \frac{1}{2k_1} (\ln(4k_1) - \ln(32k_1^3 t)), \quad (t \to \infty, \quad X \to -\infty),
\]
and the amplitude of each separated soliton is \( 2k_1 \).

Next let us investigate the relation between the obtained solution (16) and the known 2-soliton solution of the mKdV equation,
\[
u = i \left( \ln f \right)_x,
\]
\[
f = 1 + i (e^{2\xi_1} + e^{2\xi_2}) - \left( \frac{k_1 - k_2}{k_1 + k_2} \right)^2 e^{2\xi_1 + 2\xi_2}, \quad (20)
\]
where \( \xi_j (j = 12) \) is defined as Eq. (16c). We substitute
\[
\frac{\alpha_1 e^{2\xi_1^{(0)}} + \beta_1 (k_1)}{k_1 - k_2} \quad \text{and} \quad \frac{\alpha_1 e^{2\xi_2^{(0)}} + \beta_1 (k_2)}{k_2 - k_1}
\]
for \( e^{2\xi_1^{(0)}} \) and \( e^{2\xi_2^{(0)}} \), respectively, where \( \alpha_1 \) is a real constant and \( \beta_j (k_j) \) is a differentiable function of \( k_j \). So we can write Eq. (20) as
\[
f = 1 + i \alpha_1 \frac{e^{2\xi_1} - e^{2\xi_2}}{k_1 - k_2} + \left( \frac{\alpha_1}{k_1 + k_2} \right)^2 e^{2\xi_1 + 2\xi_2}, \quad (21)
\]
with new \( \xi_j \)
\[
\xi_j = k_j x - 4k_j^3 t + \beta_j (k_j) + \xi_1^{(0)}, \quad (j = 1, 2). \quad (22)
\]
Taking \( k_2 \to k_1 \) and using L’Hospital rule, Eq. (21) goes to
\[
f \to 1 + 2i \alpha_1 (x - 12k_1^2 t + \partial k_1 \beta_1 (k_1)) e^{2\xi_1} + \frac{\alpha_1^2}{4k_1^2} e^{4\xi_1}, \quad (23)
\]
which equals to Eq. (17) when \( \alpha_1 = 1/2, \beta_1 (k_1) = -(1/2) \ln 2k_1 \). That implies solutions derived by the symmetry constraint Eq. (11) are limit solutions.

4 The mKdV Equation with New Sources
Squared eigenfunctions can be used to generate sources for soliton equations.[17–8,12] The equations
\[
u_t + u_x \partial x + 6u^2 u_x + \sum_{j=1}^{N} (\phi_{1j} + \phi_{2j}) x = 0,
\]
\[
\phi_{1j,x} = -\lambda_j \phi_{1j} + u \phi_{2j},
\]

Fig. 1  Plots for \( u \) given by Eq. (7) with \( k_1 = 1.5, \xi_1^{(0)} = 0 \). (a) Shape of \( u \) for \( t = 80 \); (b) Plot 3D picture for \( u \).
\[
\phi_{2j,x} = -u\phi_{1j} + \lambda_j \phi_{2j},
\]
are called the mKdV equation with self-consistent sources (mKdVESCS). By the limit symmetry (7), we have
\[
u_t + u_{xx} + 6u^2 u_x + \sum_{j=1}^{N} (\phi_{1j}\psi_{1j} + \phi_{2j}\psi_{2j})x = 0,
\]
where \(\{k_j\}_{j=1}^{N}\) are distinct real numbers. This system is Lax integrable with the Lax pair
\[
\begin{align*}
(\phi_1)_{x} &= \left(-\lambda \ u + \mu \right) (\phi_1), \\
(\phi_1)_{t} &= \left(A \ B \right) (\phi_1),
\end{align*}
\]
where
\[
A = -4\lambda u^2 - 8\lambda^3 + \frac{1}{2} g^{-1} u \sum_{j=1}^{N} \left( \frac{2\lambda_j (\phi_{2j}\psi_{2j} - \phi_{1j}\psi_{1j})}{\lambda + \lambda_j} + \frac{\lambda (\phi_{2j}^2 - \phi_{1j}^2)}{\lambda + \lambda_j} \right) \\
+ \frac{1}{2} \partial^{-1} u \sum_{j=1}^{N} \left( \frac{2\lambda_j (\phi_{2j}\psi_{2j} - \phi_{1j}\psi_{1j})}{\lambda - \lambda_j} + \frac{\lambda (\phi_{2j}^2 - \phi_{1j}^2)}{\lambda - \lambda_j} \right),
\]
\[
B = 8\lambda^2 u - 4\lambda u_x + 2u_{xx} + u^3 - \frac{1}{2} \sum_{j=1}^{N} \left( -\frac{2\lambda_j \phi_{2j}\psi_{2j}}{\lambda + \lambda_j} - \frac{\phi_{1j}^2}{\lambda + \lambda_j} + \frac{\lambda_j \phi_{1j}^2}{(\lambda + \lambda_j)^2} \right) \\
- \frac{1}{2} \sum_{j=1}^{N} \left( \frac{2\lambda_j \phi_{2j}\psi_{2j}}{\lambda - \lambda_j} + \frac{\phi_{1j}^2}{\lambda - \lambda_j} + \frac{\lambda_j \phi_{1j}^2}{(\lambda - \lambda_j)^2} \right),
\]
\[
C = -8\lambda^2 u - 4\lambda u_x - 2u_{xx} - 4u^3 + \frac{1}{2} \sum_{j=1}^{N} \left( -\frac{2\lambda_j \phi_{2j}\psi_{2j}}{\lambda + \lambda_j} - \frac{\phi_{1j}^2}{\lambda + \lambda_j} + \frac{\lambda_j \phi_{1j}^2}{(\lambda + \lambda_j)^2} \right) \\
+ \frac{1}{2} \sum_{j=1}^{N} \left( \frac{2\lambda_j \phi_{2j}\psi_{2j}}{\lambda - \lambda_j} + \frac{\phi_{1j}^2}{\lambda - \lambda_j} + \frac{\lambda_j \phi_{1j}^2}{(\lambda - \lambda_j)^2} \right).
\]

To derive this, we use the fact
\[
L \left( \phi_{1j}^2, \phi_{2j}^2 \right) = 2\lambda_j \left( \phi_{1j}^2, \phi_{2j}^2 \right),
\]
\[
L \left( \phi_{1j}\psi_{1j}, \phi_{2j}\psi_{2j} \right) = 2\lambda_j \left( \phi_{1j}\psi_{1j}, \phi_{2j}\psi_{2j} \right) - \left( \phi_{1j}^2, \phi_{2j}^2 \right),
\]
\[
L \left( \phi_{2j}^2, \phi_{1j}^2 \right) = -2\lambda_j \left( \phi_{2j}^2, \phi_{1j}^2 \right),
\]
\[
L \left( \phi_{2j}\psi_{2j}, \phi_{1j}\psi_{1j} \right) = -2\lambda_j \left( \phi_{2j}\psi_{2j}, \phi_{1j}\psi_{1j} \right) + \left( \phi_{2j}^2, \phi_{1j}^2 \right),
\]
where
\[
L = \left( -\partial - 2u\partial^{-1}u \quad 2u\partial^{-1}u \right). \quad -2u\partial^{-1}u \quad \partial + 2u\partial^{-1}u\right),
\]
and \(\phi_j, \psi_j\) satisfy Eqs. (6) and (8). Still using the transformation (14), one can write Eq. (25) into (with \(\lambda_j = -k_j\))
\[
\begin{align*}
(D_t + D_y^2) f \cdot f &= 2i \sum_{j=1}^{N} (\bar{g}_j h_j + g_j \bar{h}_j), \\
D_x^2 \bar{f} \cdot f &= 0, \\
D_x \bar{g}_j \cdot f &= k_j g_j \bar{f}, \\
D_x \bar{h}_j \cdot f &= k_j h_j \bar{f} + g_j \bar{f},
\end{align*}
\]
(26)

This can be solved as in Sec. 3. When \(N = 1\), we have
\[
\begin{align*}
f^{(2)} &= i(x - 12k_1^2 t) e^{2\xi_1}, \\
f^{(4)} &= \frac{1}{16k_1^2} e^{4\xi_1},
\end{align*}
\]
(27a)
\[
\begin{align*}
g^{(1)}_1 &= \sqrt{\beta_1(t)} e^{\xi_1}, \\
g^{(3)}_1 &= -i \sqrt{\beta_1(t)} e^{3\xi_1} \frac{1}{4k_1},
\end{align*}
\]
(27b)
\[
\begin{align*}
h^{(1)}_1 &= \sqrt{\beta_1(t)} e^{\xi_1} (x - 12k_1^2 t), \\
h^{(3)}_1 &= i \sqrt{\beta_1(t)} \left( \frac{x}{4k_1} - 3k_1 t \right) e^{3\xi_1},
\end{align*}
\]
(27c)
\[
\begin{align*}
f^{(2j)} &= g^{(2j-1)}_1 h^{(2j-1)}_1 = 0, \\
&j \geq 3,
\end{align*}
\]
where
\[
\begin{align*}
\xi_j &= k_j x - 4k_j^3 t - \int^t_0 \beta_1(z) dz + \xi^{(0)}_j, \\
k_j, \ e^{\xi^{(0)}_j}\ are\ real\ parameters\ and \ \int^t_0 \beta_1(z) dz \ is \ arbitrary \ t-dependent \ function. \ Thus, \ a \ solution \ for \ Eq. (25) \ is \ \end{align*}
\]
\[
\begin{align*}
f &= 1 + i(x - 12k_1^2 t) e^{2\xi_1} + \frac{1}{16k_1^2} e^{4\xi_1},
\end{align*}
\]
(27d)
\[
\begin{align*}
u &= i \left( \ln \frac{f}{f} \right) = 2 \left( \text{arctan} \frac{x - 12k_1^2 t}{e^{-2\xi_1} + (1/16k_1^2) e^{2\xi_1}} \right) x,
\end{align*}
\]
(28)
where we have taken \(\varepsilon = 1\) in Eq. (15). We depict it in Fig. 2.
Fig. 2  Plots for $u$ given by Eq. (28), $k_1 = 0.5$, $\xi_1^{(0)} = 0$, $\beta_1(z) = z^2$. (a) Shape of $u$ for $t = 15$. (b) Plot 3D picture for $u$.

5 Conclusions

We have considered the limit squared-eigenfunction symmetry $\sum_{j=1}^{N}(\phi_{1j}\psi_{1j} + \phi_{2j}\psi_{2j})x$ for the mKdV equation and its applications. The obtained solution ($N = 1$ case) related to the similarity reduction is shown to be a limit solution of 2-soliton solution of the mKdV equation. Dynamics is analyzed. Asymptotically, the solution consists of two separated solitons traveling with logarithmic trajectories. Besides, we give an mKdVESCS of which the sources are related to the limit symmetry. The equation is Lax integrable and we also derive a solution via Hirota method.

References